A comparison of reference conditions in floating frame formulations that use absolute interface coordinates

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ABSTRACT

This study identifies and compares various reference conditions used to define floating frame of reference formulations in terms of absolute interface coordinates. Transforming from the generalized coordinates of the floating frame and the local elastic coordinates describing the mode shapes to absolute interface coordinates eliminates the need for Lagrange multipliers to incorporate kinematic constraints. If the reduction basis can describe rigid body motion (e.g. Craig-Bampton modes), inclusion of reference conditions which eliminate the rigid body motion from the elastic deformation is required to obtain a unique coordinate transformation. Three well-established reference conditions define the floating frame as: (1) attached to the node at the center of mass of the undeformed body, (2) attached to an interface node and (3) a weighted average of the variations of the absolute interface coordinates. The choice of reference conditions impacts the accuracy and computational cost of the resulting formulation. However, the selection is complicated by the independent derivations of the formulations and the absence of comparisons between them. In this work, the result of the derivation of the equations of motion in terms of absolute interface points are shown for each reference condition such that the formulations differ only in the implemented transformation matrices. As such, the relation between the reference conditions is illuminated. The accuracy and computational efficiency of the formulations are compared using a variation of a three-dimensional slider crank benchmark problem, providing an insight into their relative merits.

Keywords: Flexible multibody dynamics, Floating frame formulation, Reference conditions, Absolute interface coordinates.

1 INTRODUCTION

Mechanical systems composed of multiple flexible bodies are studied in the field of flexible multibody dynamics. As the bodies are connected by joints and can therefore describe large rigid body rotation with respect to each other, multibody systems are inherently nonlinear. The joints are represented by kinematic constraint equations implemented at the boundary points (also called interface points). Contrary to the movement of the global system, the flexible deformation within one body generally remains small with respect to the dimensions of the body.

The floating frame formulation is a description of flexible multibody dynamics problems which utilizes the assumption of small elastic deformations by assigning each body a floating frame which moves along with the body [1]. The absolute floating frame coordinates describe the rigid motion of each body in the system. The elastic displacement within the body is described locally with respect to the floating frame. Since the local elastic deformation is small, and therefore linear, it can be expressed as a superposition of mode shapes (extracted from the finite element model of the body) and corresponding coordinates. Reduction of the generalized coordinates can be achieved through well-established model order reduction techniques to save computation time. As a result, the number of degrees of freedom is generally small compared to other description methods for flexible multibody dynamics problems. As such, the floating frame formulation is the preferred description for multibody systems undergoing small elastic deformations [2].

A disadvantage of the floating frame formulation is the complication of the kinematic constraints due to the use of the mixed coordinate set. As the absolute boundary node coordinates are not part of the degrees of freedom of the problem, floating frame formulations require the constraints to be expressed in absolute floating frame coordinates and local elastic coordinates, resulting in nonlinear expressions. Consequently, Lagrange multipliers are required to obtain the constrained equations of motion. The resulting system of equations is of differential-algebraic form, such that time-integration scheme's suitable for differential-algebraic equations must be combined with Newton-Raphson iterations to integrate the system in time.

Alternatively, an explicit transformation of the equations of motion to absolute boundary coordinates would allow straightforward inclusion of the kinematic constraints, removing the need for Lagrange multipliers and yielding constrained equations of motion in ordinary differential form. However, to obtain this expression, transformation matrices between the absolute floating frame coordinates and local elastic coordinates and the absolute boundary coordinates must be derived. Such approaches are further referred to as absolute interface formulations, while approaches using the mixed coordinates set are further referred to as classic formulations.

To define a unique transformation, the redundancy between the three coordinate sets must be removed. The redundancy is introduced because, in the formulation as described so far, the set of local generalized coordinates are allowed to describe rigid body motion. In fact, the definition of the floating frame directly influences the amount of kinetic energy corresponding to the local displacement coordinates. Therefore, six constraints referred to as reference conditions, which remove the rigid motion from the generalized coordinates, must be introduced to obtain a unique coordinate transformation. Note that the redundancy is present in both the classic and interface formulations and must be accounted for in the classic formulations as well.

Two approaches to introduce the constraints can be found in literature. In the first approach, the constraints are introduced by choosing a set of mode shapes which cannot describe rigid body motion. By combining such mode shapes with the mean-axis or orthogonality conditions, which result from minimization of the kinematic energy corresponding to the local displacement coordinates, simplified Tisserand axes can be obtained [3, 4]. Reference conditions based on the mode shape choice are typically developed with the aim of optimizing the classic formulation.

In the second approach, the reference conditions are six position level constraints between the absolute floating frame coordinates and absolute interface coordinates which remove the rigid motion from the local deformation component. In contrast to the first approach, research into this type of reference conditions has led to the development of new formulations. These reference conditions can be characterized into two categories: body-attached frames and moving frames [5, 6]. In body-attached frame approaches, the floating frame is attached to a material point on the body. A frequently employed body-attached approach is to define the floating frame as attached to one of the boundary nodes, as developed by Cardona [7]. To prevent undesired distinction between the boundary nodes, Schilder and Ellenbroek [2] developed a formulation in which the floating frame is attached to a non-interface node. Preferably, the floating frame is placed in the center of mass of the undeformed body. In the moving frame approach, the floating frame is defined as a function of the absolute interface coordinates and therefore does not correspond to a material point. Cardona [8] defined the floating frame as a weighted average of the variations of the absolute boundary coordinates. The weights are a function of the proximity of the interface nodes to the center of mass. Each of the three described reference conditions introduces a different relationship between the coordinate sets and therefore yields a different transformation matrix.

The use of the Craig-Bampton modes [9] as model reduction basis is a common choice in absolute interface formulations. The static Craig-Bampton modes correspond to unit displacement of the boundary nodes. The additional internal modes are the eigenmodes of the constrained body, corresponding to zero displacement at the boundary nodes. Therefore, the use of Craig-Bampton modes simplifies the relation between the generalized coordinates of the floating frame and local elastic coordinates and the absolute interface coordinates. However, combinations of the static Craig-Bampton modes can describe rigid body motion and therefore must be combined with reference conditions of the second approach.

The decision of which reference condition to implement is typically left to the experience of the reader [6]. Comparing the reference conditions is troublesome as the derivations of the formulations were performed using differing approaches and notations. To the best of the author's knowledge, no comparison between the accuracy of the reference conditions is available. Therefore, this work presents a comparison between the three reference conditions, which might benefit the reader in making a substantiated choice between the formulations. To this aim, the reference conditions are considered as constraints on the elastic behaviour wherever possible, and the transformation matrices are derived accordingly. The equations of motion in boundary points are derived and differ only in the transformation matrices. In this work, only the derivation results are shown, while a future publication will present the exact derivations. To compare the accuracy and computational efficiency of the methods, the formulations are implemented on a benchmark problem.

The remainder of this paper is structured as follows; section 2 presents the equations of motion in absolute interface coordinates and the transformation matrices for the three reference conditions. Section 3 discusses the expected difference in computational efficiency and stability. In section 4, the derived methods are compared using a numerical validation problem.

2 DERIVATION RESULTS

In this section, first the derivation of the equations of motion for classic formulations is discussed. Secondly, the transformation matrices for each reference condition are presented. Then, the transformation matrices are utilized to obtain the equations of motion in absolute interface coordinates.

2.1 Equations of motion in absolute floating frame and local generalized coordinates

Consider an arbitrary body in a space described by a global Euclidean coordinate system connected to point P_O, as illustrated in Figure 1a. The body contains a material point i, to which a frame is connected at point P_i. The position vector $\boldsymbol{r}_i^{OO}$ and rotation matrix $\boldsymbol{R}_i^O$ denote the position and orientation of the material point relative to the global frame. The index notation of the position vector $\boldsymbol{r}_i^{OO}$ should be read as the position of P_i (subscript) relative to P_O (second superscript) expressed in the global frame (first superscript). Similarly, the rotation matrix $\boldsymbol{R}_i^O$ is read as the orientation of the frame attached to P_i (subscript) relative to the global frame (superscript).

To exploit the assumption of small elastic deformations, the trajectories of all material points on the body are decomposed into a large global rigid body motion and small local elastic components. To this end, a floating frame $\{P_j\}$ is introduced. One can define the position of P_i as the sum of the global position of the floating frame $\boldsymbol{r}_j^{OO}$ and the local position of the material point $\boldsymbol{r}_i^{jj}$

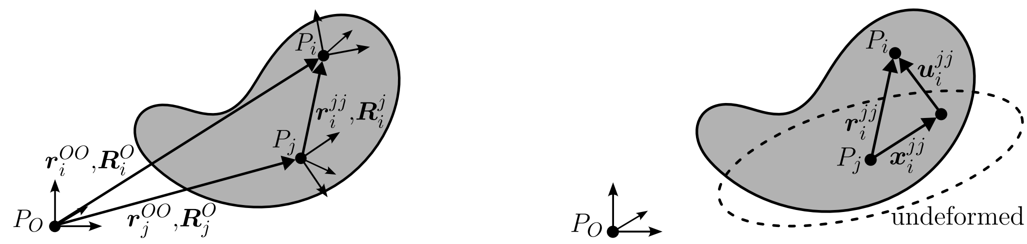
$$\boldsymbol{r}_i^{OO} = \boldsymbol{r}_j^{OO} + \boldsymbol{R}_j^O \boldsymbol{r}_i^{jj} \tag{1}$$

as illustrated in Figure 1a. The local position vector $\boldsymbol{r}_i^{jj}$ can be partitioned as the position of the material point P_i on the undeformed body $\boldsymbol{x}_i^{jj}$ and the elastic displacement $\boldsymbol{u}_i^{jj}$

$$\boldsymbol{r}_i^{jj} = \boldsymbol{x}_i^{jj} + \boldsymbol{u}_i^{jj}. \tag{2}$$

This is illustrated in Figure 1b, where the dotted shape indicates the undeformed body. Note that the undeformed body is defined in accordance with the floating frame. Therefore, $\boldsymbol{x}_i^{jj}$ remains constant over time. As the elastic deformation can generally be assumed small, it can be expressed as a linear combination of deformation shapes

$$\boldsymbol{u}_i^{jj} = \boldsymbol{\Phi}_i \boldsymbol{\eta} \quad \text{with} \quad \boldsymbol{\Phi}_i = \begin{bmatrix} \phi_1(\boldsymbol{x}_i^{jj}) & \phi_2(\boldsymbol{x}_i^{jj}) & \dots & \phi_M(\boldsymbol{x}_i^{jj}) \end{bmatrix} \tag{3}$$



(a) An illustration of the position vectors and rotation matrices used to describe material point i.

(b) The partitioning of the local position vector of P_i in an undeformed and an elastic component.

Figure 1: A schematic illustration of the definition of the coordinates.

with $\boldsymbol{\Phi}_i$ being a set of M deformation shapes evaluated at $\boldsymbol{x}_i^{jj}$ and $\boldsymbol{\eta}$ being the corresponding time dependent coordinates.

The generalized coordinates $\boldsymbol{q}$ used to describe the total system are the combination of the absolute floating frame coordinates $\boldsymbol{q}_j^{OO}$ and the local elastic coordinates $\boldsymbol{\eta}$

$$\boldsymbol{q} = \begin{bmatrix} \boldsymbol{q}_j^{OO} \\ \boldsymbol{\eta} \end{bmatrix} \quad \text{with} \quad \boldsymbol{q}_j^{OO} = \begin{bmatrix} \boldsymbol{r}_j^{OO} \\ \boldsymbol{\pi}_j^{OO} \end{bmatrix} \tag{4}$$

with $\boldsymbol{\pi}_j^{OO}$ denoting the rotation angles of the floating frame with respect to the global frame. By combining the time derivatives of the kinematic relations (1), (2) and (3) with the principle of virtual work, one can obtain the equations of motion in terms of the generalized coordinates. A detailed derivation of this is described in [2], with the resulting equation of motion

$$\left[\hat{\boldsymbol{R}}_j^O\right] \boldsymbol{M}^j \left[\hat{\boldsymbol{R}}_O^j\right] \ddot{\boldsymbol{q}} + \left[\hat{\boldsymbol{R}}_j^O\right] \boldsymbol{C}^j \left[\hat{\boldsymbol{R}}_O^j\right] \dot{\boldsymbol{q}} + \boldsymbol{K}^j \boldsymbol{q} = \boldsymbol{Q}^O \tag{5}$$

with $\boldsymbol{M}^j$, $\boldsymbol{C}^j$ and $\boldsymbol{K}^j$ being the local mass matrix, velocity dependent matrix and stiffness matrix respectively, obtained using a linear finite element model of the body. Furthermore, $\boldsymbol{Q}^O$ denotes the external forces and moments and $\left[\hat{\boldsymbol{R}}_j^O\right]$ is the block diagonal matrix $\texttt{diag}(\boldsymbol{R}_j^O, \boldsymbol{R}_j^O, \boldsymbol{1})$ with $\boldsymbol{1}$ being an identity matrix.

2.2 Transformation to absolute interface coordinates

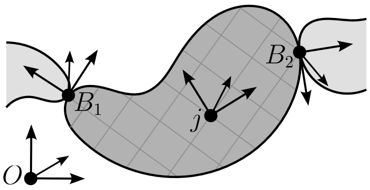
Due to the use of the mixed coordinate set, the equations of motion (5) combined with the kinematic constraint relations form a non-linear system. Thus, the system must be formulated using Lagrange multipliers and requires specialized integration techniques [10]. Alternatively, the equations of motion can be expressed in absolute interface coordinates $\boldsymbol{q}_B^{OO}$ containing the coordinates of all boundary nodes relative to the global frame. The transformation of the equations of motion to absolute interface coordinates $\boldsymbol{q}_B^{OO}$ allows for simple assimilation of the constraints and results in a system of ordinary differential equations. The transformation is particularly advantageous in combination with Craig-Bampton modes, due to the straightforward relation between the local elastic coordinates and absolute interface coordinates.

To achieve the aforementioned transformation, an explicit relation between the absolute floating frame coordinates, local elastic coordinates and absolute interface coordinates must be obtained. However, as combinations of the Craig-Bampton modes can describe rigid body motion, reference conditions which introduce six constraints between the coordinate sets are required to remove the rigid motion from the elastic deformation. To find the transformation matrices, the variation of (1) and of the rotations

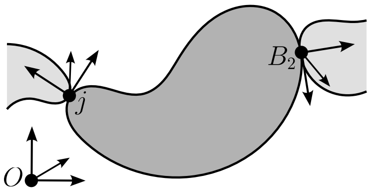
$$\delta \boldsymbol{q}_B^{jj} = \left[\bar{\boldsymbol{R}}_O^j\right] \delta \boldsymbol{q}_B^{OO} - \boldsymbol{\Phi}_{rig,B} \left[\boldsymbol{R}_O^j\right] \delta \boldsymbol{q}_j^{OO} \quad \text{with} \quad \left[\bar{\boldsymbol{R}}_j^O\right] = \texttt{diag}(\underbrace{\left[\boldsymbol{R}_j^O\right], \ldots, \left[\boldsymbol{R}_j^O\right]}_{N \text{ times}}) \tag{6}$$

must be combined with the constraints imposed by the reference conditions. Here, $\boldsymbol{\Phi}_{rig,B}$ denotes the rigid body modes evaluated at the boundary nodes, $\left[\boldsymbol{R}_j^O\right]$ is the block diagonal matrix $\texttt{diag}(\boldsymbol{R}_j^O, \boldsymbol{R}_j^O)$ and N equals the number of interface points. δ denotes a variation and $\boldsymbol{q}_B^{jj}$ contains the coordinates of all boundary nodes relative to the floating frame.

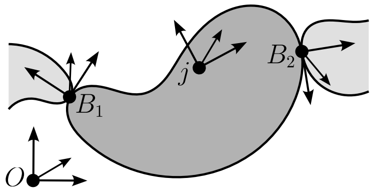
As previously introduced, three well-established reference conditions of this type define the floating frame as: (a) attached to a node close to the center of mass of the undeformed body, (b) attached to a boundary node and (c) a weighted average of the motion of the boundary nodes. Figure 2 illustrates these reference conditions, with the colored shapes representing bodies which are connected at the interface points B_1 and B_2. The derivation of the transformation matrices for each reference condition will be discussed sequentially below.



(a) The floating frame attached to the node in the center of mass of the undeformed body, with the grey lines indicating a mesh.



(b) The floating frame attached to the left interface point B_1. For clarity, the frame corresponding to B_1 is not shown.



(c) The floating frame defined as a weighted average of the variations of the absolute interface coordinates.

Figure 2: A schematic illustration of the reference conditions.

Firstly, we consider the reference condition as proposed by Ellenbroek and Schilder in [2] and illustrated in Figure 2a, in which the floating frame is attached to the node in the center of mass of the undeformed body. The rigid body motion is removed from the elastic component by constraining the elastic deformation in the floating frame to zero

$$\boldsymbol{\Phi}_{sCB,j}\delta\boldsymbol{q}_B^{jj} + \boldsymbol{\Phi}_{iCB,j}\delta\boldsymbol{\eta} = \mathbf{0}. \tag{7}$$

Here, $\boldsymbol{\Phi}_{sCB,j}$ and $\boldsymbol{\Phi}_{iCB,j}$ are the static and internal Craig-Bampton modes evaluated at the floating frame respectively. The transformation matrices are derived by combining constraint (7) with (6)

$$\begin{bmatrix}\delta\boldsymbol{q}_j^{OO}\\ \delta\boldsymbol{q}_B^{jj}\\ \delta\boldsymbol{\eta}\end{bmatrix} = \begin{bmatrix}\left[\boldsymbol{R}_j^O\right]\left(\boldsymbol{\Phi}_{sCB,j}\boldsymbol{\Phi}_{rig,B}\right)^{-1}\boldsymbol{\Phi}_{sCB,j}\left[\bar{\boldsymbol{R}}_O^j\right] & \left[\boldsymbol{R}_j^O\right]\left(\boldsymbol{\Phi}_{sCB,j}\boldsymbol{\Phi}_{rig,B}\right)^{-1}\boldsymbol{\Phi}_{iCB,j}\\ \left(\mathbf{1}-\boldsymbol{\Phi}_{rig,B}\left(\boldsymbol{\Phi}_{sCB,j}\boldsymbol{\Phi}_{rig,B}\right)^{-1}\boldsymbol{\Phi}_{sCB,j}\right)\left[\bar{\boldsymbol{R}}_O^j\right] & -\boldsymbol{\Phi}_{rig,B}\left(\boldsymbol{\Phi}_{sCB,j}\boldsymbol{\Phi}_{rig,B}\right)^{-1}\boldsymbol{\Phi}_{iCB,j}\\ \mathbf{0} & \mathbf{1}\end{bmatrix}\begin{bmatrix}\delta\boldsymbol{q}_B^{OO}\\ \delta\boldsymbol{\eta}\end{bmatrix} \tag{8}$$

with $\mathbf{0}$ being a zero matrix and $\mathbf{1}$ being an identity matrix.

Secondly, consider the reference condition as proposed by Cardona in [7] and illustrated in Figure 2b in which the floating frame is defined as attached to a boundary node. As this can be achieved by constraining the elastic deformation in the floating frame, this reference condition can be recognised as a special case of the previous derivation. Let the floating frame be attached to boundary node i from the total set of boundary nodes $1,\dots,i,\dots,N$. The constraint which defines zero elastic deformation at B_i is

$$\boldsymbol{\Phi}_{sCB,B_i}\delta\boldsymbol{q}_B^{jj} = \mathbf{0} \tag{9}$$

with $\boldsymbol{\Phi}_{sCB,B_i}$ being the static Craig-Bampton modes evaluated at boundary node i. As the static Craig-Bampton modes correspond to unity displacement at the boundary nodes, one can write

$$\boldsymbol{\Phi}_{sCB,B_i} = \begin{bmatrix}\mathbf{0} & \dots & \mathbf{1} & \dots & \mathbf{0}\end{bmatrix} \tag{10}$$

corresponding to the set of boundary nodes $1,\dots,i,\dots,N$. By combining the reference condition (9) with (6), the transformation matrices corresponding to this condition can be derived as

$$\begin{bmatrix} \delta\boldsymbol{q}_j^{OO} \\ \delta\boldsymbol{q}_B^{jj} \\ \delta\boldsymbol{\eta} \end{bmatrix} = \begin{bmatrix} \left[\boldsymbol{R}_j^O\right]\begin{bmatrix}\boldsymbol{0} & \dots & \boldsymbol{1} & \dots & \boldsymbol{0}\end{bmatrix}\left[\bar{\boldsymbol{R}}_O^j\right] & \boldsymbol{0} \\ \left(\boldsymbol{1} - \boldsymbol{\Phi}_{rig,B}\begin{bmatrix}\boldsymbol{0} & \dots & \boldsymbol{1} & \dots & \boldsymbol{0}\end{bmatrix}\right)\left[\bar{\boldsymbol{R}}_O^j\right] & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{1} \end{bmatrix} \begin{bmatrix} \delta\boldsymbol{q}_B^{OO} \\ \delta\boldsymbol{\eta} \end{bmatrix}. \tag{11}$$

Finally, we consider the reference condition as proposed by Cardona in [8] and illustrated in Figure 2c in which the floating frame is taken as a weighted average of the variations of the absolute interface coordinates $\delta\boldsymbol{q}_{B_i}^{OO}$. Since the floating frame is not attached to a node, the reference condition is not enforced by constraining the elastic deformation, but by the relation imposed by the weighted average

$$\delta\boldsymbol{q}_j^{OO} = \sum_{i=1}^{N} w_i \delta\boldsymbol{q}_{B_i}^{OO} \tag{12}$$

with w_i being the weight assigned to the i^{th} boundary node. The weights are defined based on the position of the interface points with respect to the center of mass. Combining (12) with (6) allows one to derive the transformation matrices

$$\begin{bmatrix} \delta\boldsymbol{q}_j^{OO} \\ \delta\boldsymbol{q}_B^{jj} \\ \delta\boldsymbol{\eta} \end{bmatrix} = \begin{bmatrix} \left[\boldsymbol{R}_j^O\right]\left[\boldsymbol{w}\right]\left[\bar{\boldsymbol{R}}_O^j\right] & \boldsymbol{0} \\ \left(\boldsymbol{1} - \boldsymbol{\Phi}_{rig,B}\left[\boldsymbol{w}\right]\right)\left[\bar{\boldsymbol{R}}_O^j\right] & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{1} \end{bmatrix} \begin{bmatrix} \delta\boldsymbol{q}_B^{OO} \\ \delta\boldsymbol{\eta} \end{bmatrix} \tag{13}$$

with

$$\left[\boldsymbol{w}\right] = \begin{bmatrix} w_1\boldsymbol{1} & w_2\boldsymbol{1} & \dots & w_N\boldsymbol{1} \end{bmatrix}. \tag{14}$$

2.3 Equations of motion in absolute interface coordinates

The transformation matrices for all three considered reference conditions (8), (11) and (13) are of the structure

$$\begin{bmatrix} \delta\boldsymbol{q}_j^{OO} \\ \delta\boldsymbol{q}_B^{jj} \\ \delta\boldsymbol{\eta} \end{bmatrix} = \begin{bmatrix} \left[\boldsymbol{R}_j^O\right]\boldsymbol{T}_{j1}\left[\bar{\boldsymbol{R}}_O^j\right] & \left[\boldsymbol{R}_j^O\right]\boldsymbol{T}_{j2} \\ \boldsymbol{T}_{B1}\left[\bar{\boldsymbol{R}}_O^j\right] & \boldsymbol{T}_{B2} \\ \boldsymbol{0} & \boldsymbol{1} \end{bmatrix} \begin{bmatrix} \delta\boldsymbol{q}_B^{OO} \\ \delta\boldsymbol{\eta} \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} \delta\boldsymbol{q}_B^{OO} \\ \delta\boldsymbol{\eta} \end{bmatrix}. \tag{15}$$

Here $\boldsymbol{T}_{j1}$ and $\boldsymbol{T}_{j2}$ transform the absolute boundary coordinates and the local elastic coordinates corresponding to the internal Craig-Bampton modes to absolute floating frame coordinates, and $\boldsymbol{T}_{B1}$ and $\boldsymbol{T}_{B2}$ transform the same coordinates to the local boundary node coordinates. Since the transformation matrices are of the same structure for each reference condition, the equations of motion in absolute interface coordinates can be obtained by substituting (15) in (5),

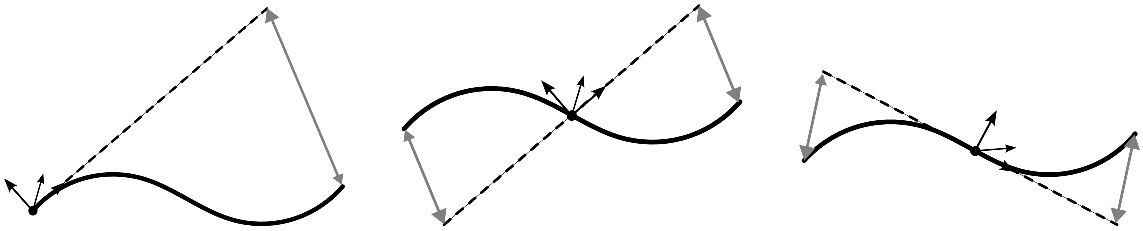
$$\boldsymbol{T}^T\boldsymbol{M}^O\boldsymbol{T}\ddot{\boldsymbol{q}}_B^{OO} + \boldsymbol{T}^T\left(\boldsymbol{M}^O\dot{\boldsymbol{T}} + \boldsymbol{C}^O\boldsymbol{T}\right)\dot{\boldsymbol{q}}_B^{OO} + \boldsymbol{T}^T\boldsymbol{K}^j\boldsymbol{q}_B^{jj} = \boldsymbol{T}^T\boldsymbol{Q}^O. \tag{16}$$

Here $\boldsymbol{M}^O$ is the global mass matrix $\left[\hat{\boldsymbol{R}}_j^O\right]\boldsymbol{M}^j\left[\hat{\boldsymbol{R}}_O^j\right]$ and $\boldsymbol{C}^O$ denotes the global velocity dependent matrix $\left[\hat{\boldsymbol{R}}_j^O\right]\boldsymbol{C}^j\left[\hat{\boldsymbol{R}}_O^j\right]$. As such, simulations using the different reference conditions can be performed solely by changing the transformation matrices.

3 DISCUSSION OF THE TRANSFORMATION MATRICES

As the differences between the formulations are due to transformation matrices, comparing them can give insight into expected computational efficiency differences. Implementing the formulations as presented so far requires one to update the rigid body modes and therefore the transformation matrices at every time step. Additionally, both the center of mass and weighted average approaches require re-evaluation of an inverse term at every time step. However, the floating frame formulation is typically used for problems in which the elastic deformation can be assumed small with respect to the dimensions of the body ($\boldsymbol{r}^{jj}_{B_i} \approx \boldsymbol{x}^{jj}_{B_i}$). Applying the assumption of small deformations results in constant transformation matrices for all three reference conditions.

It should be noted that the definition of the floating frame influences the size of the elastic deformation and therefore the validity of the assumption of small deformations. Figure 3 illustrates how the size of the largest elastic displacement varies between the reference conditions. The solid line represents the deformed body, while the frame and the dashed line represent the floating frame and the corresponding undeformed body. The grey arrows indicate the largest elastic displacement.



(a) The floating frame attached to the left boundary node.

(b) The floating frame defined as a weighted average of the variations of the absolute boundary coordinates.

(c) The floating frame attached to the node in the center of mass of the undeformed body.

Figure 3: A schematic illustration of the dependence of the largest elastic deformation (grey arrows) on the definition of the floating frame (black frame). The solid line shows the deformed body which contains a boundary node at each end. The dashed line shows the undeformed body.

Defining the floating frame as attached to an interface point is expected to result in the largest elastic deformation terms, as illustrated in Figure 3a. Therefore, this formulation is likely the first to require updating the rigid modes at every time step or refining the number of bodies used to describe the problem, both of which increase the computational cost.

In the weighted average approach, the size of the elastic deformation depends strongly on the main deformation shapes occurring in the system. For the example, while Figure 3 shows differences in elastic deformation between the weighted average and center of mass approach, a different deformation shape (e.g. a first bending mode) might yield an equally large elastic deformation for both approaches. Therefore, the validity of the small elastic deformations assumption can vary greatly between problems. Furthermore, it is important to realize that the trajectory of the floating frame has no physical meaning in the weighted average approach. In contrast, the trajectory of the floating frame in the other two approaches represents the trajectory of a node of the body.

4 VALIDATION

To investigate the accuracy of the formulations for different reference conditions, (16) is implemented on the three dimensional slider crank benchmark problem. The slider crank consists of a rigid crank, a flexible connector and a slider, as illustrated in Figure 4. The crank has a length of 0.15 m and is positioned at a distance of 0.1 m (d) along the y-axis. It is capable of rotating in the yz-plane. The connector has a length of 0.3 m and a circular cross-section with diameter of 6 mm. It is modelled with a density of 8780 kg/m^3, a Young's modulus of 200 GPa and a shear modulus of 77 GPa. The slider has a mass equal to half the weight of the connector and can slide long the x-axis only. In the initial configuration, the system is stationary and the crank is positioned parallel to the positive z-axis. The angular velocity of the crank (ω_{cr}) is linearly increased from 0 to 200 rad/s over a duration of 0.1 seconds, as illustrated in Figure 5. The normalized midpoint deflection of the connector is used to compare the simulation results. The deflection is defined as the perpendicular displacement of the midpoint of the connector with respect to the straight line

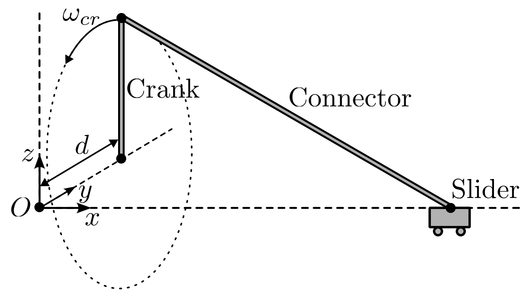


Figure 4: An illustration of the 3D slider crank system with a flexible connector.

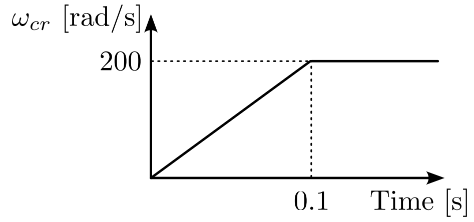


Figure 5: The prescribed angular velocity.

that connects the outer nodes of the connector and normalized by the length of the connector.

Figure 6 and 7 depict the deflection results over the first 0.1 seconds when the connector is defined using four and two bodies respectively. In both cases, no internal modes were included. For the interface point approach, the floating frame is defined in the node which couples the connector to the slider (right). The solid line in both figures represents the result of a non-linear FEM simulation[1] which can be considered as reference.

If the connector is defined by four bodies, the deflection obtained with each reference condition closely approximates the FEM simulation result. As anticipated, reducing the number of bodies to two yields less accurate deflections, which is particularly noticeable around the 0.9 seconds mark in Figure 7. For the center of mass and weighted average approaches, the result still closely represents the FEM simulation. However, for a floating frame defined in the right boundary node, the resulting deformation deviates more strongly from the reference.

In practice, refining an arbitrary body into multiple bodies is difficult. Therefore, the simulation results for a connector consisting of one body are of interest. In contrast to the simulations with multiple bodies, the static Craig-Bampton modes cannot accurately represent the dynamic behaviour when one body is used. Therefore, ten internal modes have been included to obtain the results shown in Figure 8. Further deviations from the FEM simulation are visible for all reference conditions. For the center of mass and weighted average approaches, the results still accurately represent the phase, although the amplitude starts to deviate around 0.05 seconds. However, for the interface point approach significant differences in both amplitude and phase are visible.

Based on the results, it is observed that defining the floating frame in an interface point performs least accurately, which is consistent with the expectation that the elastic deformation is the largest

[1]The simulation was performed in Abaqus 2021 HotFix 6 using the updated Lagrangian approach, 720 quadratic volume elements (quadratic bricks) and implicit Hilber-Hughes-Taylor time integration.

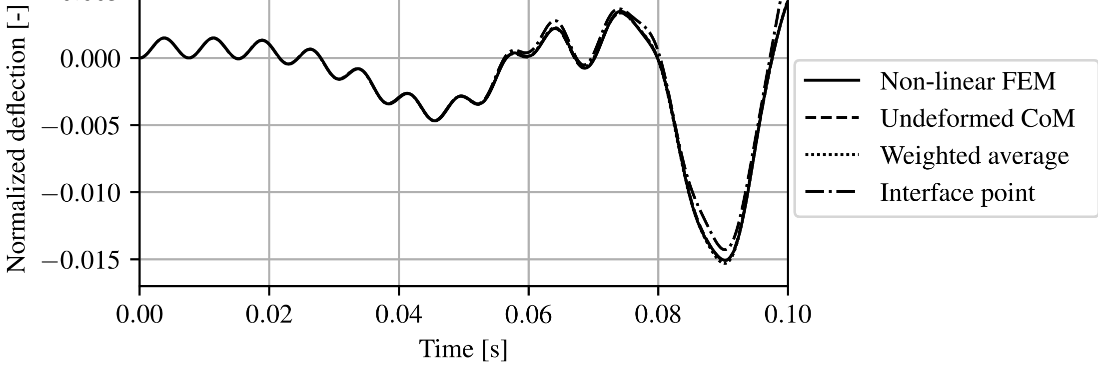


Figure 6: The normalized midpoint deflection obtained by nonlinear FEM and using each reference conditions. The connector is modelled with four bodies and no internal Craig-Bampton modes.

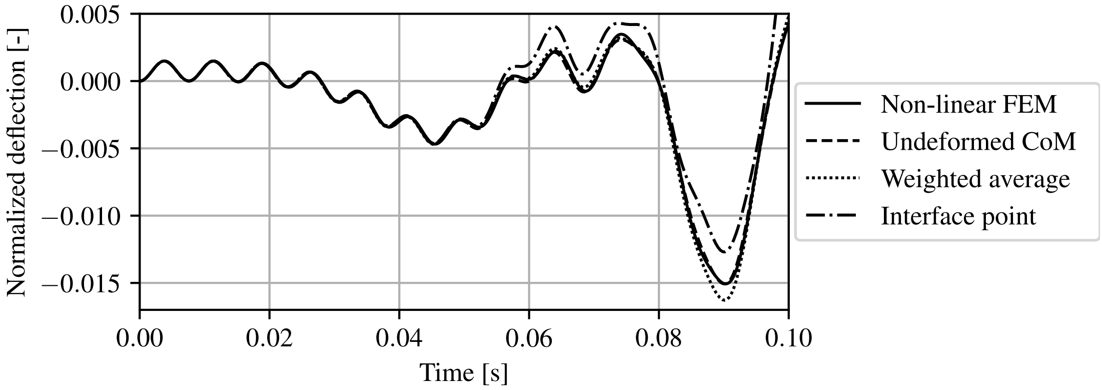


Figure 7: The normalized midpoint deflection obtained by nonlinear FEM and using each reference conditions. The connector is modelled with two bodies and no internal Craig-Bampton modes.

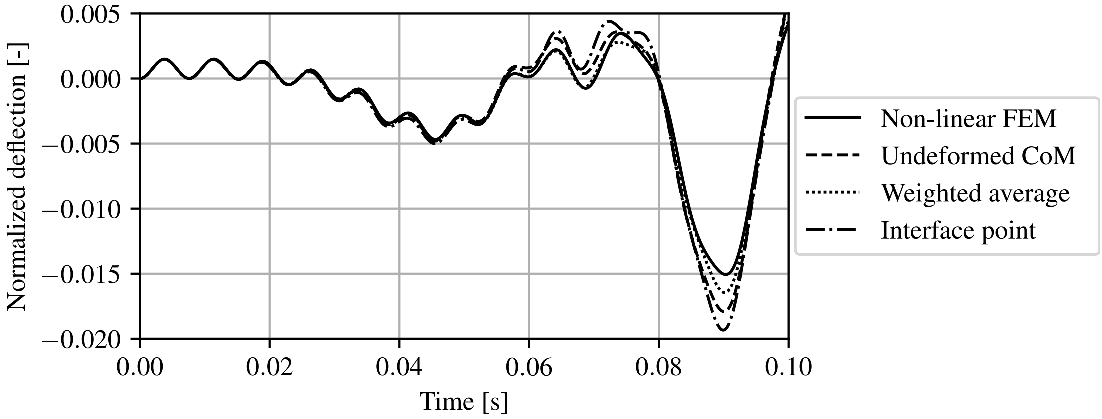


Figure 8: The normalized midpoint deflection obtained by nonlinear FEM and using each reference conditions. The connector is modelled with one body and ten internal Craig-Bampton modes.

for this reference condition. Although including internal modes is necessary for a connector modelled with one body, the simulation is still fast due to the reduction in the number of bodies. The remaining two reference conditions are comparable in terms of accuracy.

5 CONCLUSIONS

Floating frame formulations employ a mixed coordinates set comprising the absolute floating frame coordinates and local elastic coordinates. Due to the mixed coordinates, Lagrange multiplies are required to implement the constraints. Nonlinear constraint equations can be avoided by transforming the floating frame formulation to absolute boundary coordinates. However, to obtain a unique transformation, reference conditions which eliminate the rigid motion from the elastic movement are required. If the reduction set can describe rigid body motion, e.g. Craig-Bampton modes, the reference conditions take the form of six position level constraints between the absolute boundary and floating frame coordinates. Three reference conditions of this type have been considered: defining the floating frame in the node at the center of mass, defining the floating frame as attached to an interface node, and defining the floating frame as a weighted average of the variations of the absolute interface coordinates.

This work has demonstrated that for each of these reference conditions, a transformation matrix with the same general shape can be obtained. All transformation matrices are constant if small elastic deformation is assumed. The accuracy of the reference conditions has been assessed by simulating a three dimensional slider crank. The center of mass and weighted average approaches yield similar results which closely represent the reference non-linear FEM simulation. Defining the floating frame as attached to an interface point yields noticeably larger deviations from the desired result as well as phase discrepancies when the connector is modelled using one body. This behaviour is expected, as attaching the floating frame to a boundary node is likely to result in larger elastic deformations compared to the other reference conditions and thus would be the first to violate the assumption of small elastic deformations.

This work offers insight into the resemblances and disparities among the three considered reference conditions. Through careful consideration, the results indicate that the reference conditions which define the floating frame as attached to the center of mass and as weighted average of the variations of the absolute boundary coordinates are preferable to use. These reference conditions are comparable in terms of computational efficiency, but offer more accurate results compared to attaching the floating frame to a interface node.

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